

SU(2,1) Dynamics of Multiple Giant Dipole Resonance Coulomb Excitation

M.S. Hussein, A.F.R. de Toledo Piza, O.K. Vorov

Instituto de Fisica, Universidade de Sao Paulo

Caixa Postal 66318, 05315-970,

Sao Paulo, SP, Brasil

(13 August 1999)

Abstract

We construct a three-dimensional analytically soluble model of the nonlinear effects in Coulomb excitation of multiphonon Giant Dipole Resonances (GDR) based on the SU(2,1) algebra. The full 3-dimensional model predicts further enhancement of the Double GDR (DGDR) cross sections at high bombarding energies. Enhancement factors for DGDR measured in thirteen different processes with various projectiles and targets at different bombarding energies are well reproduced with the same value of the nonlinearity parameter with the exception of the anomalous case of ^{136}Xe which requires a larger value.

One of the most interesting applications of Coulomb Excitation in heavy ion collisions [1–8] is investigation of multi phonon nuclear Giant Resonances (GR) [2]. Possibility to excite multiple GR involves Bose statistics of collective excitations and constitutes the “family property” of vibrational collective motion in both finite (nuclei, clusters) and infinite quantum systems. Within this concept, making no distinction between infinite and finite systems, the excitation process can be modeled via a multidimensional quantum oscillator coupled linearly to the external time-dependent field, providing excellent agreement with the single-GR experimental data [2].

Validity of this completely linear theory has been questioned by nearly all the experimental data wherever multi-phonon GR has been observed [2]: the double Giant Dipole Resonance (GDR) excitation cross sections [8] are found about $1.3 - 2$ bigger than follows from theory. This shortcoming of the linear theory, known as “enhancement factor problem”, has been addressed widely in current literature within a number of approaches: higher order perturbation theory [9], anharmonic effects [7], [10], concept of hot phonons [11–13]. Clearly, nonlinear effects, that in principle can not be neglected in finite Fermi systems [14], are not easily dealt with either at a microscopical or even at a phenomenological level [7].

It is therefore appealing to construct a natural, soluble “minimal extension” of the harmonic model of Coulomb excitations, in which deviations from the linear scheme are reasonably described via a few parameters. Without dealing in depth with the microscopic theory, we present here such a “minimal extension”. This single-parameter nonlinear model can be solved exactly using algebraic properties of boson operator combinations forming algebra $SU(2,1)$. The model allows us to correlate all the experimental data for the “enhancement factors” in various nuclei and various bombarding energies, using a single value of the universal nonlinear parameter.

Within the semiclassical approach [15] to Coulomb excitation, the projectile motion is approximated by a constant velocity v on a straight line classical trajectory with impact parameter b and internal excitation is treated quantum mechanically. The intrinsic state $|\Psi(t)\rangle$ of the system undergoing excitation obeys the time-dependent Schrödinger equation,

$$i\partial/\partial t|\Psi(t)\rangle = [H_0 + V(t)] |\Psi(t)\rangle, \quad |\Psi(t = -\infty)\rangle = |0\rangle, \quad (1)$$

where H_0 is the intrinsic Hamiltonian and $V(t) = v_1(t)[\mathcal{D}_{-1}^\dagger - \mathcal{D}_{+1}^\dagger] + v_0(t)\mathcal{D}_0^\dagger + h.c.$ is the channel-coupling interaction with \mathcal{D}^\dagger and \mathcal{D} the dipole operators acting in the space of the multi-GDR states created by the boson operators d_m^\dagger , with the angular momentum projection m ($\hbar=c=1$). The functions $v_{1(0)}(t)$ describe the interaction with the electromagnetic field [2], [4] (see below). The excitation probability of an intrinsic state $|N\rangle$ with N phonons in a collision with impact parameter b and the total cross section σ_N are [2,4,5,15]:

$$P_N(b) = |\langle N|\Psi(t = \infty)\rangle|^2, \quad \sigma_N = 2\pi \int_0^\infty b P_N(b) db. \quad (2)$$

The internal nuclear Hamiltonian H_0 is nearly harmonic [2], [16], [17], so $H_0 = \omega N = \omega \sum_m d_m^\dagger d_m$. In principle, this does not exclude anharmonicities [18] in the transition operators $\mathcal{D}^\dagger, \mathcal{D}$ when expanded in terms of phonon operators

$$\mathcal{D}_m^\dagger = d_m^\dagger + x \sum_{m_1} d_m^\dagger d_{m_1}^\dagger d_{m_1} + \sum_{m_1} x_{m_1} d_m^\dagger d_{m_1}^\dagger d_{m_1}^\dagger + x_2 \sum_{m_1 m_2} d_m^\dagger d_{m_1}^\dagger d_{m_1} d_{m_2}^\dagger d_{m_2} + \dots, \quad (3)$$

These effective nonlinearities [14], [3] could result from perturbation theory treatment of anharmonicities in the phonon Hamiltonian, from coupling to other degrees of freedom, both collective (e.g., quadrupole GR) and noncollective [11] etc. The linear limit of the problem, $\mathcal{D}_m^\dagger = d_m^\dagger$, is exactly soluble giving the Poisson formula for the excitation probabilities

$$P_N = e^{-\rho} \frac{\rho^N}{N!}, \quad \rho = \sum_{m=0,\pm 1} |\alpha_m^{harm}|^2 = \sum_{m=0,\pm 1} \left| \int_{-\infty}^\infty v_m(t) e^{i\omega t} dt \right|^2 \quad (4)$$

where the amplitudes α_m^{harm} are given by the modified Bessel functions K_1 and K_0 [2].

In order to reduce the number of unknown parameters in Eq.(3) it is reasonable to restrict the higher-order nonlinear corrections $\propto x_i$. We save the first, dominating, nonlinear term in (3) with its coefficient $0 \leq x \ll 1$ and save only those terms in (3) which match terms appearing in the expansion of the square root, $\mathcal{D}_m^\dagger \rightarrow d_m^\dagger (1 + 2x \sum_m d_m^\dagger d_m)^{1/2}$.

The nonlinear effects are now controlled by the single parameter x . This may be justified, by noting that the leading term is saved, while the ansatz for the higher order terms, which

should be small anyway, obeys the basic requirement that they get smaller as the order increases. Further, this parametrization leads to a soluble problem.

The group theoretical solution is based on the consideration of the eight operators

$$\begin{aligned} D^+ &= (d_{-1}^+ - d_{+1}^+) \sqrt{k + N/2}, \quad D_0^+ = d_0^+ \sqrt{2k + N}, \quad J^+ = \frac{1}{2^{1/2}} (d_{-1}^+ - d_{+1}^+) d_0, \\ D^0 &= \frac{1}{4} [(d_{-1}^+ - d_{+1}^+)(d_{-1} - d_{+1}) + 2(2k + N)], \quad D_0^0 = \frac{1}{2} [2k + N + d_0^+ d_0]. \end{aligned} \quad (5)$$

and $D^- = (D^+)^\dagger$, $D_0^- = (D_0^+)^\dagger$, $J^- = (J^+)^\dagger$ with $k \equiv \frac{1}{4x}$ the Casimir invariant. They form a closed SU(2,1) algebra, the non-compact analogue of SU(3). The pairwise commutators between (5) can be evaluated directly: Three of them are $[D^-, D^+] = 2D^0$, $[D_0^-, D_0^+] = 2D_0^0$, $[J^-, J^+] = 2(D_0^0 - D^0)$. The other nonzero ones are $[D^-, D^0] = 2[D^-, D_0^0] = [D_0^-, J^-] = D^-$; $[D_0^-, D_0^0] = 2[D_0^-, D^0] = [D^-, J^+] = D_0^-$, and $[D^-, D_0^+] = 2[J^-, D^0] = -2[J^-, D_0^0] = J^-$. The remaining nonzero commutators are given by Hermitean conjugates to the above. In the interaction representation, the evolution equation $i \frac{\partial}{\partial t} |\psi(t)\rangle = e^{iH_0 t} V(t) e^{-iH_0 t} |\psi(t)\rangle$ (1) with (3) reads:

$$i(\partial/\partial t) |\psi(t)\rangle = 2x^{1/2} [\tilde{v}_1(t) D^\dagger + \tilde{v}_1^*(t) D^- + \tilde{v}_0(t) D_0^\dagger + \tilde{v}_0^*(t) D_0^-] |\psi(t)\rangle. \quad (6)$$

where $\tilde{v}_1 \equiv v_1 e^{i\omega t}$ and $\tilde{v}_0 \equiv v_0 e^{i\omega t} / \sqrt{2}$. Any product of exponentials involving the operators from the set (5) can be reduced to a simpler exponential (see, e.g., [19]), as due to closure of the pairwise commutators between (5), no new operator structures arise while re-arranging order of the operators in the time-ordered exponential (see, e.g., [20]) that solves (1),(6)

$$|\psi(t)\rangle \equiv T \exp \left\{ \int_{-\infty}^t e^{iH_0 \tau} V(\tau) e^{-iH_0 \tau} d\tau \right\} |0\rangle = e^{aD^+ + bD_0^+} e^{fJ^+} e^{cD^0 + dD_0^0} e^{gJ^-} e^{a'D^- + b'D_0^-} |0\rangle \quad (7)$$

where the eight time-dependent c-numbers, the Latins $a - g$, must be chosen so $|\psi(t)\rangle$ (7) obeys the Schrödinger equation (6). The ansatz (7) can be simplified using $D^- |0\rangle = D_0^- |0\rangle = J^\pm |0\rangle = 0$; $D^0 |0\rangle = D_0^0 |0\rangle = k |0\rangle$, which follow from (5). The expression (7) then reduces to

$$|\psi(t)\rangle = \left[1 - 4x (|\alpha(t)|^2 + |\beta(t)|^2) \right]^{\frac{1}{4x}} e^{i\phi(t)} e^{\frac{\alpha(t)}{\sqrt{k}} D^+ + \frac{\beta(t)}{\sqrt{k}} D_0^+} |0\rangle, \quad (8)$$

where the first factor comes from unitarity, $\langle \psi(t) | \psi(t) \rangle = 1$. The phase $\phi(t)$ is unimportant for the following. Substituting (8) into (6), we obtain, after some lengthy algebra using

heavily the operator identity $e^A B e^{-A} = B + [A, B] + (1/2!)[A, [A, B]] + \dots$, the two nonlinear equations for the amplitudes α and β

$$i\partial\alpha/\partial t - Q\alpha = \tilde{v}_1(t), \quad i\partial\beta/\partial t - Q\beta = \tilde{v}_0(t), \quad (9)$$

with $Q \equiv 4x[\tilde{v}_0^*(t)\beta + \tilde{v}_1^*(t)\alpha]$ and the initial condition $\alpha(-\infty) = \beta(-\infty) = 0$. After projecting the asymptotic state (8) at $t = \infty$ onto the states with definite number of GDR phonons, we obtain the non-Poissonian expression for the excitation probabilities

$$P_N = \frac{\Gamma(\frac{1}{2x} + N) \left[4x \left(|\bar{\alpha}|^2 + |\bar{\beta}|^2\right)\right]^N}{N! \Gamma(\frac{1}{2x}) \left[1 - 4x \left(|\bar{\alpha}|^2 + |\bar{\beta}|^2\right)\right]^{-\frac{1}{2x}}}. \quad (10)$$

where $\bar{\alpha}$ and $\bar{\beta}$ are the asymptotic solutions of the system (9) at $t = \infty$. Their values can be easily tabulated by solving (9). If $x \rightarrow 0$, Eq.(10) is reduced to Poisson, while the solutions to (9) are reduced to the harmonic amplitudes (4), $|\alpha| \rightarrow |\alpha_1^{harm}| = \frac{2F\xi^2}{\omega} K_1(\xi)$ and $|\beta| \rightarrow |\alpha_0^{harm}|/\sqrt{2} = \frac{2F\xi^2}{\gamma\omega} K_0(\xi)$. Thus, the harmonic results are restored. At nonzero nonlinearity $x > 0$, the multiple GDR excitation probabilities P_N turn out to be enhanced as compared to their values P_N^{harm} in the harmonic limit, Eq.(4). The cross sections σ_N and their harmonic values, σ_N^{harm} are given by integrating in Eq.(2) from the grazing value $1.2(A_{ex}^{1/3} + A_{sp}^{1/3})$. The enhancement factors $r_N = \sigma_N/\sigma_N^{harm}$ can be studied and compared with the data. The functions $v_m(t)$, in the notations $\xi = \frac{\omega b}{v\gamma}$, $\tau = \frac{\omega t}{\xi}$ are [4,5]

$$v_1(t) = Ff^3, \quad v_0(t) = F\sqrt{2}\gamma \left[-\frac{\partial f}{\partial \tau} - i\xi v^2 f \right], \quad F = \frac{Z_{sp}e^2\gamma}{2b^2} \left[\frac{N_{ex}Z_{ex}}{A_{ex}^{2/3} m_N \cdot 80 MeV} \right]^{\frac{1}{2}}. \quad (11)$$

where $f = (1 + \tau^2)^{-\frac{1}{2}}$ and $\gamma = (1 - v^2)^{-\frac{1}{2}}$. In the strength F , m_N and e are the proton mass and charge, Z , N and A denote the nuclear charge, the neutron number and the mass numbers, the labels ex (sp) refer to the excited (spectator) nucleus in the colliding pair.

Let us discuss the energy dependence of the two-phonon enhancement factor r_2 . In Ref. [21], we have studied this behavior for the Pb+Pb system at the bombarding energies in range 70–700 MeV·A, using truncated (two-dimensional) dynamics and neglecting the longitudinal response in the excitation process ($v_0 \equiv 0$). Within this “toy model” [21], the dynamics is

described by the three operators D, D^+ and D^0 which form the $SU(1,1)$ subalgebra of the $SU(2,1)$. The enhancement factor drops steadily as the bombarding energy grows. This is not the case *at higher energies*, when the transverse approximation breaks down and full solution based on the $SU(2,1)$ algebra is required. This is illustrated in Fig.1a, where the enhancement factor r_2 is shown as a function of γ for the case of $Pb + Pb$ collision in the energy range up to 4 Mev, as compared to the “transverse approximation”.

The remarkable fact about the full 3-dimensional model is that the enhancement factor r_N starts to grow again as the relativistic factor γ passes the “extremal” point ($\sim 1.3-1.5$ for heavy colliding nuclei). Thus the 3-dimensional $SU(2,1)$ model predicts a new interesting qualitative effect which can be tested in experiments using higher bombarding energies. This behavior of r_N is related to the γ -dependence of the “scaled” longitudinal function \tilde{v}_0 (Eq.11) [4,5]: $\tilde{v}_0(t) = v_0(t)e^{i\omega t} = F\sqrt{2} \left[-\gamma \frac{d}{d\tau} (f e^{i\xi\tau}) + i \frac{\xi}{\gamma} f e^{i\xi\tau} \right]$. The γ -dependence of the two terms here is very different. While the first term scales as γ , the absolute maximum of the second one scales as $\xi/\gamma = \gamma^{-2}$. The first term is proportional to the time derivative of the function $\frac{e^{i\omega t}}{(1+\tau^2)^{3/2}}$ which dies out at $t=\pm\infty$. Therefore, *in the harmonic* limit, when the longitudinal amplitude β reduces to the simple integral over time of the function \tilde{v}_0 , the first term simply vanishes. The harmonic solution is then given by the integral of the second term to give $\propto \frac{1}{\gamma} K_0(\xi)$ which dies out as $1/\gamma$ as bombarding energy grows, and the longitudinal response becomes negligible at high bombarding energies as is well known [2-4]. This is not the case *at nonzero nonlinearity* $x \neq 0$: the amplitude β is given by the solution to the coupled nonlinear system (9), and the first term in \tilde{v}_0 not only does contribute but in fact becomes dominant. Unlike the second term, the first one grows with γ and this determines the behavior of the enhancement factor.

We present below in Figs.1b, Fig.2 and Fig.3a exact numerical results for the cross sections calculated according to Eqs.(2), and (10) and using numerical solution of Eq.(9). Since the nonlinear parameter x is an internal property of the nucleus in which the GDR is excited, it is reasonable to expect that it varies from one nuclear species to another. For the sake of comparison with the experimental data, it is expedient however to choose a single

average value $x = 0.29$ for all cases and use it to calculate the cross sections. As seen in Figs.1a,2 and Fig.3a, the nonlinear model can in this way (no fitting) reproduce rather well the experimental values [8] of r_2 for the twelve excitation processes of Pb (Fig.2a,b) and Au (Fig.2c), using different bombarding energies and targets/projectiles. An exceptional case is Xe, where considerably larger value of x is required. To illustrate this, the optimal values of x for each individual datum are shown in Fig.3b, together with the adopted average value.

In conclusion, we presented a simple soluble model to account for the nonlinear effects in the transition operators for the Coulomb excitation of multi-phonon GDR via relativistic heavy ion collisions. The solubility of the model is based on the group theoretical properties of the boson operators. It allows to construct the solution for the dynamics of the multi-phonon excitation within the coupled-channel approach. The well known harmonic phonon model appears to be a limiting case of the present model when the nonlinearity goes to zero. The main advantages of the limiting harmonic case (unrestricted multiphonon basis, preservation of unitarity and possibility of analytical treatment) remain present in our nonlinear scheme. Therefore, the model can be viewed as a natural extension of the harmonic phonon model to include the nonlinear effects in a consistent way while keeping the model solvable. With proper modifications, it can be used in other (nuclear, molecule *etc.*) problems.

At low enough bombarding energies, the enhancement factor drops as the bombarding energy grows. This is consistent with the data and gives results similar to those recently obtained in a different context, with a theory based on the concept of fluctuations (damping) and the Brink-Axel mechanism [11], [12]. The interesting property of the full three-dimensional results obtained here is that the enhancement factor starts growing again at high bombarding energies ($\sim 0.3\text{--}0.5\text{GeV}\cdot\text{A}$ for heavy colliding nuclei.) Besides being an interesting prediction to be tested in experiment, this behavior allows one to separate the effects of nonlinearity considered here and the effects proposed in works [11–13].

The work has been supported by the FAPESP and by the CNPq.

REFERENCES

- [1] P.F.Bortignon, A.Braco and R.A.Brogia, *Giant Resonances, Nuclear Structure at Finite Temperatures*, (Harwood Acad., New York, 1998).
- [2] H. Emling, Prog. Part. Nucl. Phys. **33**, 729 (1994).
- [3] T. Aumann, P.F. Bortignon, and H. Emling, Annu. Rev. Nucl. Part. Sci. **48**, 351 (1998).
- [4] C. A. Bertulani *et al*, Phys.Rev. **C53**, 334 (1996); C. A. Bertulani, nucl-ex/9803009.
- [5] C. A. Bertulani and V. Yu. Ponomarev, Phys.Rep.,in press.
- [6] G. Baur and C.A. Bertulani, Phys. Lett. **B174**, 23 (1986); P.G. Hansen, Nucl. Phys. **A553**, 89c (1993);J. Norbury and G. Baur, Phys. Rev. **C48**, 1915 (1993).
- [7] P. Chomaz and N. Frascaria, Phys. Rep. **252**, 275 (1995).
- [8] R. Schmidt et al. Phys. Rev. Lett. **70**, 1767 (1993); J.L.Ritman et al. Phys. Rev. Lett. **70**, 533,2659 (1993); T.Aumann et al. Phys. Rev. **C47**, 1728 (1993); K.Boretzky et al., Phys. Lett. **B384**, 30 (1996); J.R. Beene, Nucl. Phys. **A569**, 163c (1994).
- [9] C.A. Bertulani and V. Zelevinsky, Phys. Rev. Lett. **71**, 967 (1993).
- [10] P.F. Bortignon and C.H.Dasso, Phys. Rev. **C56**, 574 (1997).
- [11] B.V.Carlson *et al.*, Ann. Phys. (N.Y.), 1999, in press.
- [12] B.V.Carlson, M. S. Hussein, and A. F. R. de Toledo Piza, Phys. Lett. **B431**, 249 (1998).
- [13] B.V.Carlson *et al.*, Phys. Rev. **C60**, 014604 (1999); B.V.Carlson *et al.*, Phys.Rev. **C59**, 2689 (1999); B.V.Carlson and M.S.Hussein, Phys.Rev. **C59**, R2343 (1999).
- [14] S.T. Belyaev and V.G. Zelevinsky, Nucl.Phys. **39**, 582 (1962).
- [15] K. Alder and A. Winther, Coulomb Excitation (Academic Press, New York, 1965).
- [16] F.Catara, Ph. Chomaz and N. Van Giai, Phys. Lett. **B233**, 6 (1989).

- [17] G. F. Bertsch and H. Feldmeier, Phys.Rev. **C56**, 839 (1997).
- [18] D.M. Brink, A.F.R. de Toledo Piza, and A.K.Kerman, Phys.Lett. **19**, 413 (1965).
- [19] J.N. Ginocchio and G. Wenes, in: *Relations between Structure & Reactions in Nuclear Physics*, Ed. D.H.Feng, M.Vallieres and B.H.Wildenthal, (World Scientific, Singapore, 1987) p.53; Y.Alhassid, F.Gürsey and F.Iachello, Phys. Rev. Lett. **50**, 873 (1983).
- [20] H. Kleinert, Fortschr. Phys. 26: **11**, 565 (1978).
- [21] M. S. Hussein, A. F. R. de Toledo Piza and O. K.Vorov, Phys. Rev. **C59**,R1242 (1999).

Figure Captions

Fig.1. (a) The enhancement factor r_2 as a function of γ for $^{208}\text{Pb}+^{208}\text{Pb}$ collision. The 3D results vs. the transverse approximation results (SU(1,1) toy model, [21]) for $x = 0.19$.

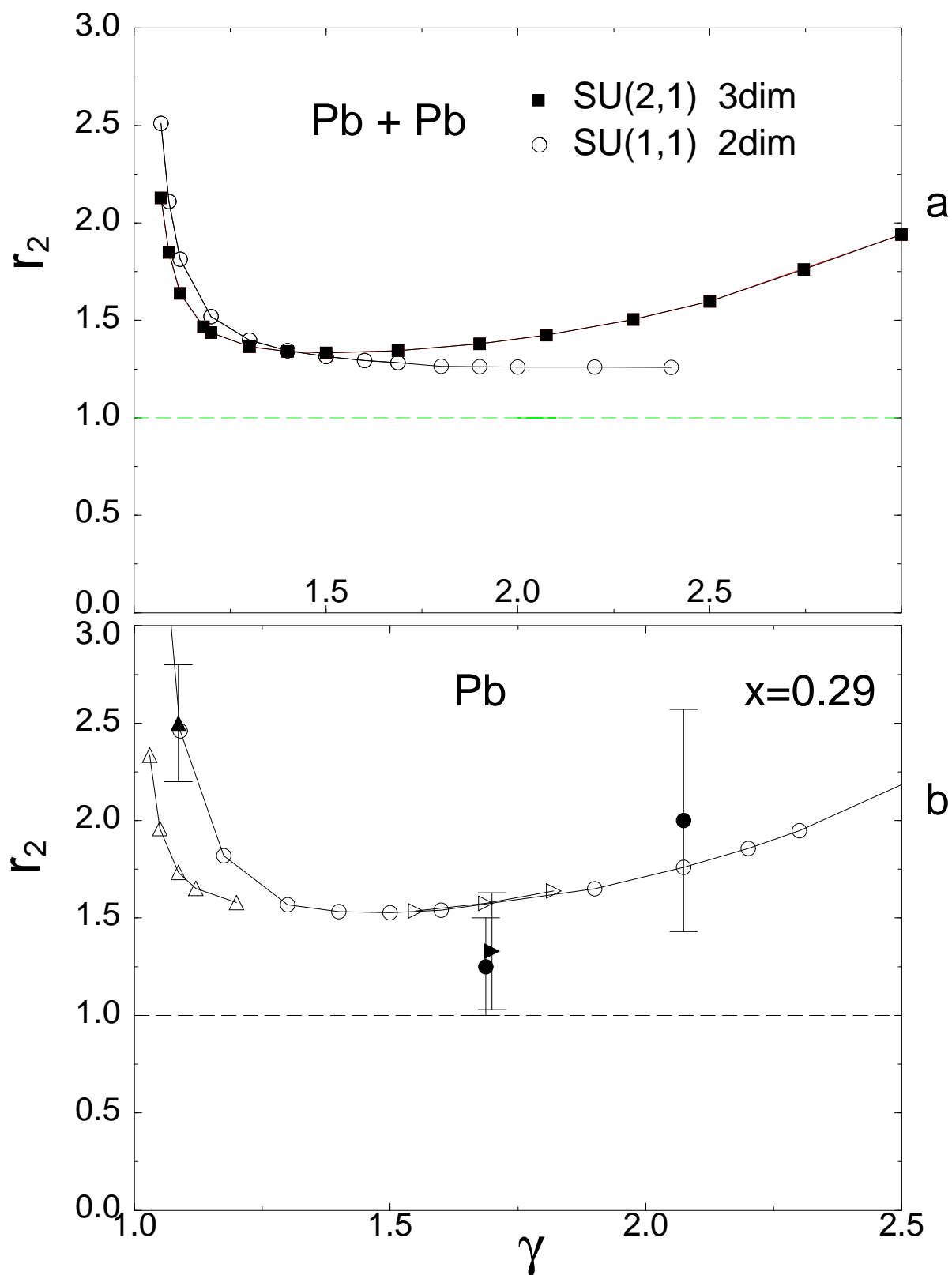
(b) Enhancement factor $r_2 = \sigma_2/\sigma_2^{harm}$ for the DGDR excitation in ^{208}Pb as a function of γ (open symbols connected by a line). The data [8] (filled symbols): triangle up - Zn projectile, triangle right - U tagret, circles - Pb target and projectile. **The value of the nonlinear parameter is kept fixed, $x = 0.29$.**

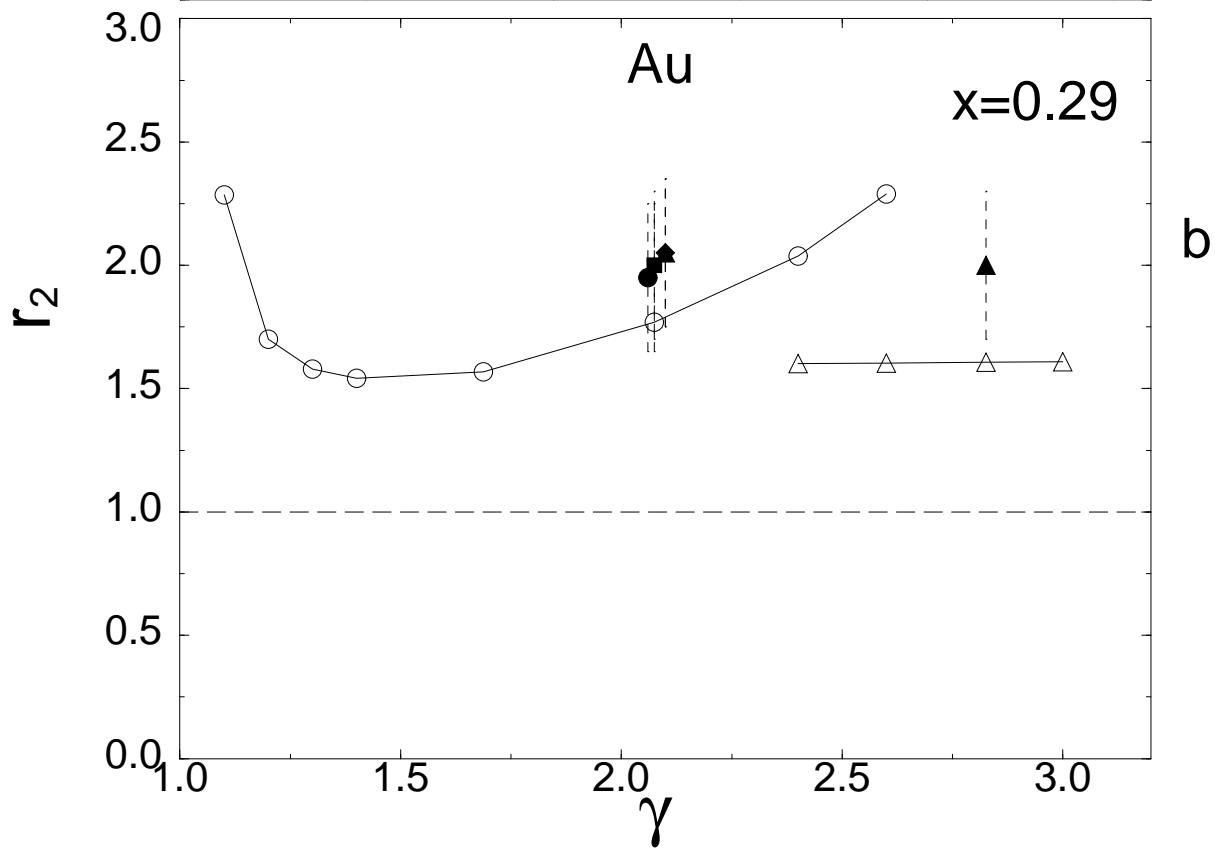
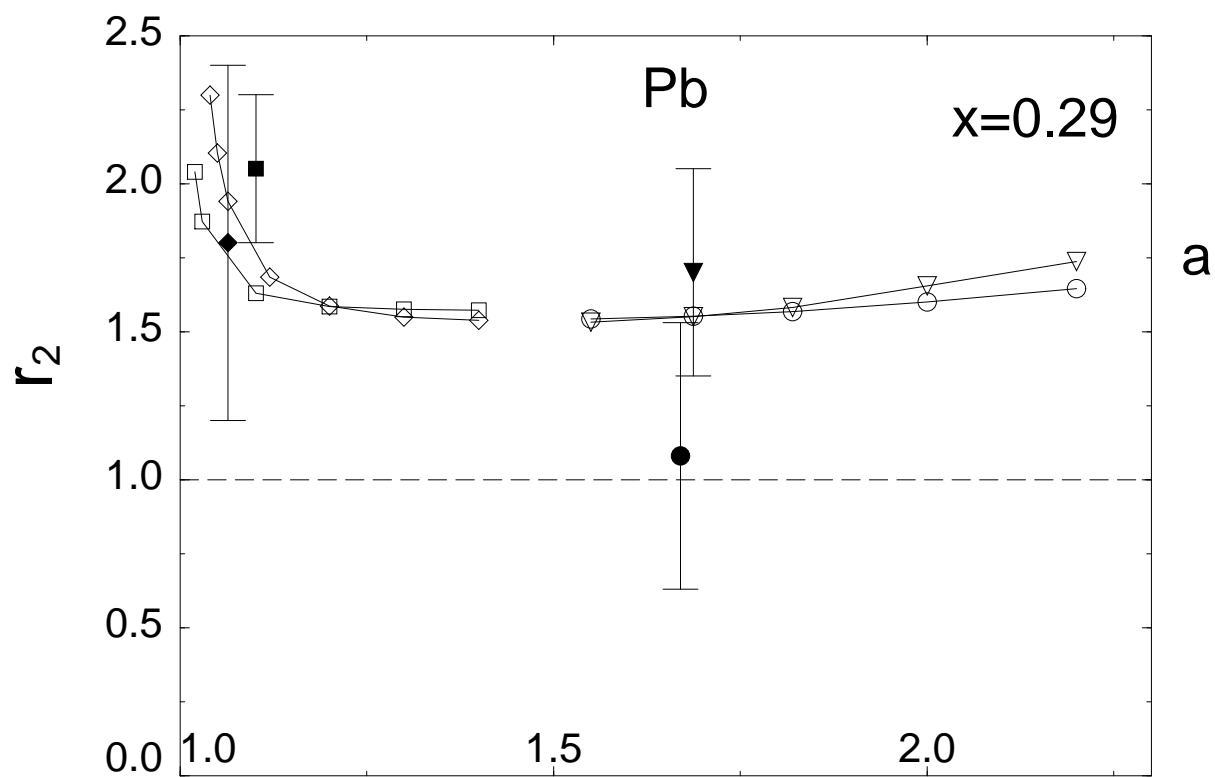
Fig.2. (a) The same as in Fig.1b, $x = 0.29$. Square - Ar projectile, diamond - Kr projectile, triangle down - Ho, and circle - Sn. (b) The same for the DGDR in Au. Circle - Kr projectile, square - Au projectile, diamond - Bi projectile and triangle - Ne projectile. Open circles - theoretical values for the Bi projectile (the results for Kr and Au projectiles are the same at $\gamma \sim 2$).

Fig.3. (a) The same as in Fig.1b, Fig2, but for the DGDR in Xe , $x = 0.29$.

(b) The optimal values of the nonlinear parameter x for the thirteen processes vs strength parameter F/ω , Eq.(11). The processes are numbered in the order they appear in Figs.1b, 2a, 2b, 3a (from left to right).

Hussein Fig. 1





Hussein Fig.3

